

# Higgs/EW Fit at future $e^+e^-$ colliders

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with much thanks to the Snowmass SMEFT  
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Jorge has now described the general outline and our results for the all-inclusive SMEFT global fit. This involves a very large number of operator coefficients that must be determined from data. The full picture is very complex.

In this talk, I would like to discuss a subset of this problem, the determination of SMEFT coefficients at future e+e- colliders.

There are several advantages in working in this restrictive context:

Only a subset of dimension-6 SMEFT operators are relevant. If we deal with electroweak processes, we can restrict ourselves to operators that appear at the tree level.

These operators are associated with specific leptons or bosons that can be directly identified experimentally.

e+e- Higgs factories supply a large number of independent observables to constrain SMEFT corrections. This is especially true for linear colliders with beam polarization.

These points lead to a tight closed system of SMEFT coefficients, whose values can be obtained robustly without degenerate directions.

At  $e^+e^-$  Higgs factories, **it is possible to use SMEFT as a model**. Such a model is needed, e.g., to determine the Higgs boson width and the absolute normalization of Higgs boson couplings. For this task, we need a **maximally model-independent description**. SMEFT actually provides this, with only the assumption that the new particles are much heavier than the Higgs boson mass.

To make my point, I will include in the fits in this talk only the minimal amount of information from sources other than an e<sup>+</sup>e<sup>-</sup> Higgs factory. The uncertainties can be somewhat improved by including HL-LHC results and some results from lower energies. Here I will include only

precision electroweak inputs:  $\alpha(m_Z)$ ,  $G_F$ ,  $G_{F\tau}$

HL-LHC expected ratios of BRs:

$$BR(H \rightarrow \gamma\gamma)/BR(H \rightarrow 4\ell)/BR(H \rightarrow \mu^+\mu^-)$$

I will develop the analysis in 3 stages:

1. Include only **purely bosonic SMEFT operators and Higgs couplings**, plus lepton-W,Z couplings assuming lepton universality
2. Remove the assumption of **lepton universality**
3. Add **4-fermion contact interactions**

In this talk, I will not discuss the addition of CP-violating operators. Also, adding the (t,b) system adds a very large number of operators. For the processes that I will consider, these are not important for  $E_{CM} \lesssim 500$  GeV. The effects of these operators will be discussed in Victor's talk.

Let's begin with 3 subsets of the full set of SMEFT operators:

Operators with  $\gamma$ ,  $W$ ,  $Z$ ,  $h$  only (using equations of motion to minimize this set. There are **7** of these:

$$\begin{aligned} & \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi) (\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 \\ & + \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W_{\mu\nu}^a W^{a\mu\nu} + \frac{4gg' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W_{\mu\nu}^a B^{\mu\nu} \\ & + \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W_{\mu\nu}^a W^{b\nu}{}_\rho W^{c\rho\mu} \end{aligned}$$

Operators that modify the couplings of leptons to SM bosons—**3** of these. (Here I assume lepton universality.)

$$\begin{aligned} & + i \frac{c_{HL}}{v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi) (\bar{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi) (\bar{L} \gamma_\mu t^a L) \\ & + i \frac{c_{HE}}{v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi) (\bar{e} \gamma_\mu e) . \end{aligned}$$



Also we need **2** more linear combinations of these operators that modify  $\Gamma_W, \Gamma_Z$ .

Operators that modify individual Higgs boson couplings (**5** of these).

$$\sum_f \frac{c_f}{v^2} |\Phi|^2 L_f^\dagger \cdot \Phi f_R + h.c. + \frac{c_{GG}}{v^2} \Phi^\dagger \Phi G_{\mu\nu}^a G^{a\mu\nu}$$

Put  $c_6$  aside for a moment. Then we have 16 Wilson coefficients plus **4** tree-level SM parameters. In the ILC fits, we also add **2** parameters to represent the branching ratios to invisible and visible but exotic final states.

In all, we have 22 parameters.

At 250 GeV, there are not 22 independent observables in Higgs processes alone. However, the SMEFT effective Lagrangian is the effective Lagrangian for **all of particle physics**. So we can get additional observables from precision electroweak and  $e^+e^- \rightarrow W^+W^-$ . The result is a highly overdetermined fit.

In fact, even  $G_F$  is a redundant piece of information. This will be important later.

|                    | ILC250 | no $G_F$ | +no $\Gamma_Z$ |
|--------------------|--------|----------|----------------|
| $g(HWW)$           | 0.48   | 0.49     | 0.50           |
| $g(HZZ)$           | 0.49   | 0.50     | 3.00           |
| $g(Hbb)$           | 0.99   | 0.99     | 1.01           |
| $g(Hcc)$           | 1.82   | 1.82     | 1.83           |
| $g(Hgg)$           | 1.61   | 1.62     | 1.63           |
| $g(H\tau\tau)$     | 1.13   | 1.13     | 1.15           |
| $g(H\gamma\gamma)$ | 1.09   | 1.09     | 3.08           |
| $\Gamma_h$         | 2.30   | 2.30     | 2.34           |

## Treatment of $e^+e^- \rightarrow W^+W^-$

The triple gauge boson coupling measurement has been studied at Higgs factories in full simulation only at 500 GeV, and with linear collider polarized beams.

The cross section also depends on SMEFT coefficients, e.g. those contributing to the electron W and Z couplings, but this was not taken into account.

To treat this, we use the “optimal observables” method (Diehl, Nachtmann, Gunion, Grzadkowski, He).

A fully differential cross section is given in terms of SMEFT parameters  $c_j$  by

$$\frac{d\sigma}{d\Omega} = \left. \frac{d\sigma}{d\Omega} \right|_{SM} + \sum_j c_j \frac{d\bar{\sigma}_j}{d\Omega}$$

Then the statistical error on the  $c_j$  is given by a covariance matrix given by

$$(\mathbf{C}^{-1})_{jk} = \int d\Omega \frac{(d\bar{\sigma}_j/d\Omega) (d\bar{\sigma}_k/d\Omega)}{d\sigma/d\Omega|_{SM}} \cdot \mathcal{L} \cdot \epsilon$$

$\epsilon$  is the efficiency for events to be included in the analysis.

This is evaluated by Monte Carlo integration. As many parameters as you wish can be easily added.

In principle, the cross section should be smeared by detector effects and  $\epsilon$  should depend on the point in phase space.

In practice, we take the tree-level, parton-level cross section and, after simple angular cuts, assume that  $\epsilon$  is constant over phase space.  $\epsilon$  is chosen to agree with ILC full simulation, we find  $\epsilon \sim 45\%$

Since  $e^+e^- \rightarrow W^+W^-$  is also used to determine the luminosity and effective beam polarization, we include these as nuisance variables and integrate them out. This degrades the TGC determination by about 10%.

The model is simple, but it can be used for any energy and polarization setting.

Next, relax the assumption of lepton universality.

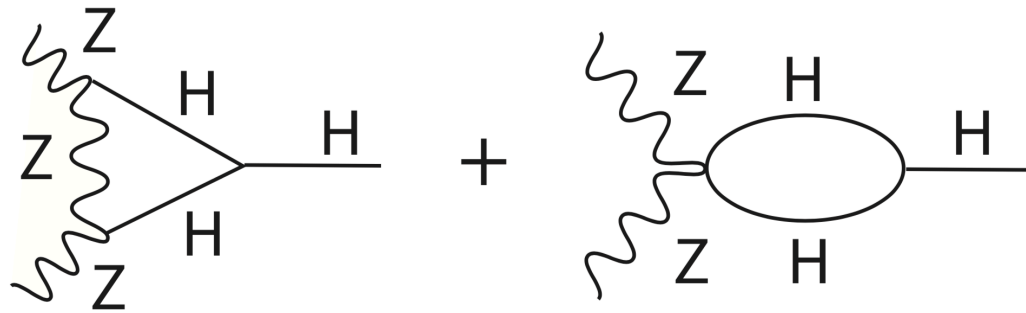
The observables  $\Gamma(Z \rightarrow \ell^+ \ell^-)$ ,  $A_\ell$ ,  $BR(W \rightarrow \ell \nu)$  are measured independently for each lepton. So, we can extend the fit from 3 to 9 Higgs-lepton coupling parameters.

This leads to a 28-parameter SMEFT fit, and the results are robust with respect to this extension.

| ILC250             | original (16+6) | all leptons (22+6) |
|--------------------|-----------------|--------------------|
| $g(HWW)$           | 0.48            | 0.45               |
| $g(HZZ)$           | 0.49            | 0.45               |
| $g(Hbb)$           | 0.99            | 0.97               |
| $g(Hcc)$           | 1.82            | 1.81               |
| $g(Hgg)$           | 1.61            | 1.61               |
| $g(H\tau\tau)$     | 1.13            | 1.11               |
| $g(H\gamma\gamma)$ | 1.09            | 1.07               |
| $\Gamma_h$         | 2.30            | 2.27               |
| $c_{HLe}$          | 0.024           | 0.020              |
| $c'_{HLe}$         | 0.024           | 0.027              |
| $c_{HEe}$          | 0.035           | 0.023              |
| $c_{HL\mu}$        |                 | 0.034              |
| $c'_{HL\mu}$       |                 | 0.046              |
| $c_{HL\mu}$        |                 | 0.036              |
| $c_{HL\tau}$       |                 | 0.040              |
| $c'_{HL\tau}$      |                 | 0.045              |
| $c_{HL\tau}$       |                 | 0.037              |

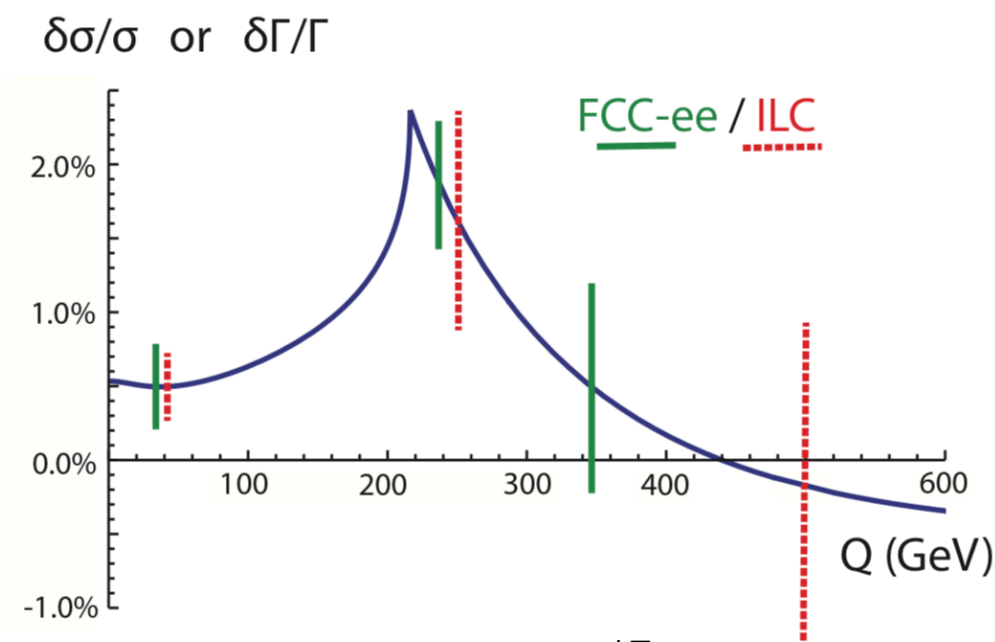


Now add back  $c_6$  .  $c_6$  affects the single-Higgs processes through a loop correction to the  $hWW$  and  $hZZ$  vertices.



McCulloch

At any one energy, this effect is highly degenerate with other parameters that shift the normalization of these vertices. Running at two energies is needed to make the effect visible. But, in that case, the fit is robust with respect to the addition of  $c_6$  .



| ILC500             | 16+6 | w. $c_6$ |     |
|--------------------|------|----------|-----|
| $g(HWW)$           | 0.34 | 0.36     |     |
| $g(HZZ)$           | 0.34 | 0.38     |     |
| $g(Hbb)$           | 0.58 | 0.59     |     |
| $g(Hcc)$           | 1.17 | 1.17     |     |
| $g(Hgg)$           | 0.95 | 0.96     |     |
| $g(H\tau\tau)$     | 0.74 | 0.76     |     |
| $g(H\gamma\gamma)$ | 1.00 | 1.01     |     |
| $\Gamma_h$         | 1.55 | 1.60     |     |
| $c_6$              | —    | 0.54     | 54% |

For FCC-ee, the corresponding error on  $c_6$  from this “indirect” determination is 48%.

Finally, add  $e^+e^- \rightarrow f\bar{f}$  processes. These add 5 SMEFT operators for each flavor (3 parameters for e):

$$\begin{aligned} & \frac{c_{LL}}{v^2} (\bar{L}_e \gamma^\mu L_e) (\bar{L}_f \gamma_\mu L_f) + \frac{4c'_{LL}}{v^2} (\bar{L}_e \gamma^\mu t^a L_e) (\bar{L}_f \gamma_\mu t^a L_f) \\ & + \frac{c_{LR}}{v^2} (\bar{L}_e \gamma^\mu L_e) (\bar{f}_R \gamma_\mu f_R) + \frac{c'_{RL}}{v^2} (\bar{e}_R \gamma^\mu e_R) (\bar{L}_f \gamma_\mu L_f) \\ & + \frac{c_{RR}}{v^2} (\bar{e}_R \gamma^\mu e_R) (\bar{f}_R \gamma_\mu f_R) \end{aligned}$$

With polarized beams, 4 parameters are determined by the total cross section, the FB asymmetry, and the polarization asymmetries in these. The 5th parameter is determined, e.g., by  $G_F$ .

With unpolarized beams, there are only 2+1 observables, so the fit cannot determine all of the parameters independently. Using multiple energies can help.

The sensitivity to individual operators is very high. For the compositeness scales  $\Lambda$  , expected 95% CL exclusion limits are

|            | $\Lambda_{LL}$ | $\Lambda_{RR}$ | $\Lambda_{VV}$ | $\Lambda_{AA}$ |
|------------|----------------|----------------|----------------|----------------|
| $f = e$    |                |                |                |                |
| ILC500     | 171            | 165            | 268            | 167            |
| CEPC 360   | 70             | 73             | 147            | 82             |
| $f = \mu$  |                |                |                |                |
| ILC500     | 136            | 135            | 200            | 179            |
| CEPC 360   | 70             | 73             | 147            | 82             |
| $f = \tau$ |                |                |                |                |
| ILC500     | 135            | 134            | 198            | 175            |
| CEPC 360   | 67             | 64             | 111            | 98             |

(The PDG defines compositeness interactions as current-current interactions with the prefactor  $2\pi/\Lambda^2$  .)

Yong will discuss  $e^+e^-q\bar{q}$  contact interactions.

Finally, the SMEFT fit allows us to assess the tradeoff between luminosity and polarization for circular vs. linear colliders. I will show results from the same fitter with the same inputs, but with the specific run plans of each collider.

All results here come from the 35+6 parameter fit.

# First, results at 240/250 GeV:

|                      | ILC/C <sup>3</sup><br>2 ab <sup>-1</sup> , 80/30 | FCC-ee<br>5 ab <sup>-1</sup> , 0/0 | CEPC<br>20 ab <sup>-1</sup> , 0/0 |
|----------------------|--|------------------------------------|-----------------------------------|
| Higgs couplings (%): |  |                                    |                                   |
| $g(hWW)$             | 0.45   | 0.49                               | 0.35                              |
| $g(hZZ)$             | 0.44   | 0.48                               | 0.34                              |
| $g(hbb)$             | 0.97   | 0.75                               | 0.45                              |
| $g(hgg)$             | 1.61   | 1.15                               | 0.63                              |
| $g(hcc)$             | 1.80   | 1.27                               | 0.69                              |
| $g(h\tau\tau)$       | 1.11   | 0.84                               | 0.50                              |
| $g(h\mu\mu)$         | 3.96   | 3.77                               | 3.05                              |
| $g(h\gamma\gamma)$   | 1.06   | 1.05                               | 0.94                              |
| $\Gamma_h$           | 2.27   | 1.70                               | 1.00                              |
| invis.               | 0.36   | 0.38                               | 0.35                              |
| unclass.             | 1.60   | 1.13                               | 0.68                              |

|             | ILC/C <sup>3</sup><br>2 ab <sup>-1</sup> , 80/30 | FCC-ee<br>5 ab <sup>-1</sup> , 0/0 | CEPC<br>20 ab <sup>-1</sup> , 0/0 |
|-------------|--|------------------------------------|-----------------------------------|
| TGCs (%)    |  |                                    |                                   |
| $g_{1Z}$    | 0.158  | 0.155                              | 0.140                             |
| $\kappa_A$  | 0.097  | 0.096                              | 0.075                             |
| $\lambda_A$ | 0.132  | 0.151                              | 0.127                             |

# next, results for the full program

| add $\rightarrow$    | ILC/C <sup>3</sup><br>4 ab <sup>-1</sup> @ 500 | FCC-ee<br>1.5 ab <sup>-1</sup> @ 360 | CEPC<br>1 ab <sup>-1</sup> @ 360 |
|----------------------|--|--------------------------------------|----------------------------------|
| Higgs couplings (%): |  |                                      |                                  |
| $g(hWW)$             | 0.34   | 0.36                                 | 0.29                             |
| $g(hZZ)$             | 0.34   | 0.36                                 | 0.29                             |
| $g(hbb)$             | 0.58   | 0.63                                 | 0.42                             |
| $g(hgg)$             | 0.95   | 0.97                                 | 0.59                             |
| $g(hcc)$             | 1.17   | 1.12                                 | 0.66                             |
| $g(h\tau\tau)$       | 0.74   | 0.72                                 | 0.46                             |
| $g(h\mu\mu)$         | 3.76   | 3.68                                 | 0.30                             |
| $g(h\gamma\gamma)$   | 1.00   | 1.00                                 | 0.92                             |
| $\Gamma_h$           | 1.54   | 1.44                                 | 1.00                             |
| invis.               | 0.32   | 0.33                                 | 0.32                             |
| unclass.             | 1.20   | 0.97                                 | 0.63                             |

| add $\rightarrow$          | ILC/C <sup>3</sup><br>4 ab <sup>-1</sup> @ 500 | FCC-ee<br>1.5 ab <sup>-1</sup> @ 360 | CEPC<br>1 ab <sup>-1</sup> @ 360 |
|----------------------------|--|--------------------------------------|----------------------------------|
| triple gauge couplings (%) |  |                                      |                                  |
| $g_{1Z}$                   | 0.0834   | 0.127                                | 0.0913                           |
| $\kappa_A$                 | 0.0315   | 0.0520                               | 0.0251                           |
| $\lambda_A$                | 0.0806   | 0.116                                | 0.113                            |

finally, results for 4-fermion operators from the fit including all (almost all) operators:

| add $\rightarrow$                 | ILC/C <sup>3</sup><br>4 ab <sup>-1</sup> @ 500 | FCC-ee<br>1.5 ab <sup>-1</sup> @ 360 | CEPC<br>1 ab <sup>-1</sup> @ 360 |
|-----------------------------------|--|--------------------------------------|----------------------------------|
| compositeness scale limits (TeV): |  |                                      |                                  |
| $c_{LL}(ee)$                      | 134  | 56                                   | 65                               |
| $c_{LR}(ee)$                      | 139  | —                                    | —                                |
| $c_{RR}(ee)$                      | 38   | 20                                   | 25                               |
| $c_{LL}(e\mu)$                    | 33   | 6                                    | 4                                |
| $c'_{LL}(e\mu)$                   | 33   | 31                                   | 31                               |
| $c_{LR}(e\mu)$                    | 123  | —                                    | —                                |
| $c_{RL}(e\mu)$                    | 125  | —                                    | —                                |
| $c_{RR}(e\mu)$                    | 135  | 6                                    | 3                                |
| $c_{LL}(e\tau)$                   | 12   | 6                                    | 6                                |
| $c'_{LL}(e\tau)$                  | 12   | 11                                   | 11                               |
| $c_{LR}(e\tau)$                   | 117  | —                                    | —                                |
| $c_{RL}(e\tau)$                   | 118  | —                                    | —                                |
| $c_{RR}(e\tau)$                   | 129  | 5                                    | 6                                |



It is remarkable that, at (polarized)  $e^+e^-$  Higgs factories, it is possible to robustly independently determine all dimension-6 SMEFT coefficients that enter the key processes at the tree level.

This will give a great deal of specificity to the identification of any deviation from the Standard Model.